EARTH — MOON — EARTH COMMUNICATION

HIGH ACCURACY COMPUTING OF DOPPLER-FIZEAU EFFECT

Former approximate algorithm

High accuracy
Doppler-Fizeau effect algorithm

Polarization effect

Bessel's computing method
(short description and graphical explanation)

Some typical Doppler-Fizeau effect curves

Parabolic antenna built by the author. Diameter = 10.50 m

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Reims, France, May 2004, revised January 2015

HIGH ACCURACY COMPUTING OF DOPPLER-FIZEAU EFFECT

*by Franck Tonna, F5SE**

INTRODUCTION

EME operation at and above 5 GHz requires precise computing of Doppler-Fizeau effect, if the EME operator wants to know with accuracy on which frequency, either to find his own echo, or to find his "Moon partner".

At the beginning of the EME activity, on the lower bands (144 and 432 MHz), only a coarse empirical knowledge of frequency shift was sufficient to observe EME signals. This shift was (and always is) low, and signals fell (and always fall) within the filter bandwidth of the receiving set-up. In those early days, Moon position was even computed "by hand", with the help of data published in various astronomical almanacs, interpolation formulas and a great deal of patience... Doppler-Fizeau effect computing was a "deluxe feature" only possible with computers, devices which, in those days, were out of range for the average EME enthusiast.

Later, when access to computing facilities of any kind became a reality, some EME operators started writing software which not only were able to give Moon position, but also were able to give an acceptable approximation of the Doppler-Fizeau effect for the lower bands.

With the increase of EME activity on the microwave bands, it is now necessary to have a better approach of the Doppler-Fizeau effect, because the amount of frequency shift can be too large for receiving set-up bandwidths. Echoes may fall back far outside of the filter, without proper re-tuning.

Following this introduction, three diagrams explain the geometry of the four speed components involved in accurate Doppler-Fizeau effect computing, as shown in decreasing order of significance:

1) Earth's rotation speed component (primary component).

2) Moon's motion along its orbit (primary component).

3) Moon's right ascension speed component (secondary component).

4) Moon's declination speed component (secondary component).

Both secondary components are a consequence of the parallax effect.

The first part of this paper shortly describes one of some early algorithms developed by the author, some 25 years ago. This short algorithm only consisted in two simple formulas, which only occupied a few lines in any software code. But its accuracy is far from being satisfying at and above 5 GHz.

The second part describes in detail the full algorithm developed in order to obtain high precision computing of the Doppler-Fizeau effect. This algorithm contains no less than 50 formulas. Rectangular coordinates and matrix processing are widely used, which significantly simplify code writing and increase computing speed, and there are no approximations anymore. The accuracy is only limited by computing truncation, by the precision of the Moon motion theory... and in case of real time computing, by the built-in clock/timer of the computer (proper time settings, drift, etc.). According to 10 GHz EME operators already using this algorithm, echo frequency falls ±20 Hz from the computed "target" frequency, in most cases these ±20 Hz being the resolution of the receiving set-up.

The third part is a "side drop" of the algorithm. By just adding a few more formulas, it is also possible to easily compute the "polarization angle" between two stations, as it would be seen by a hypothetical observer "riding" on the Earth–Moon axis (see details in that part).

The Moon motion theory used here is the theory called *ELP2000−85*, written in English by two French scientists, in collaboration with scientists from JPL and from many other astronomy research centers around the world (*Lunar Tables and Programs from 4000 B.C. to A.D. 8000*, by Michelle Chapront-Touzé and Jean Chapront, Willman-Bell, Inc. PO Box 35025, Richmond VA, USA. ISBN: 0–943396–33–6).

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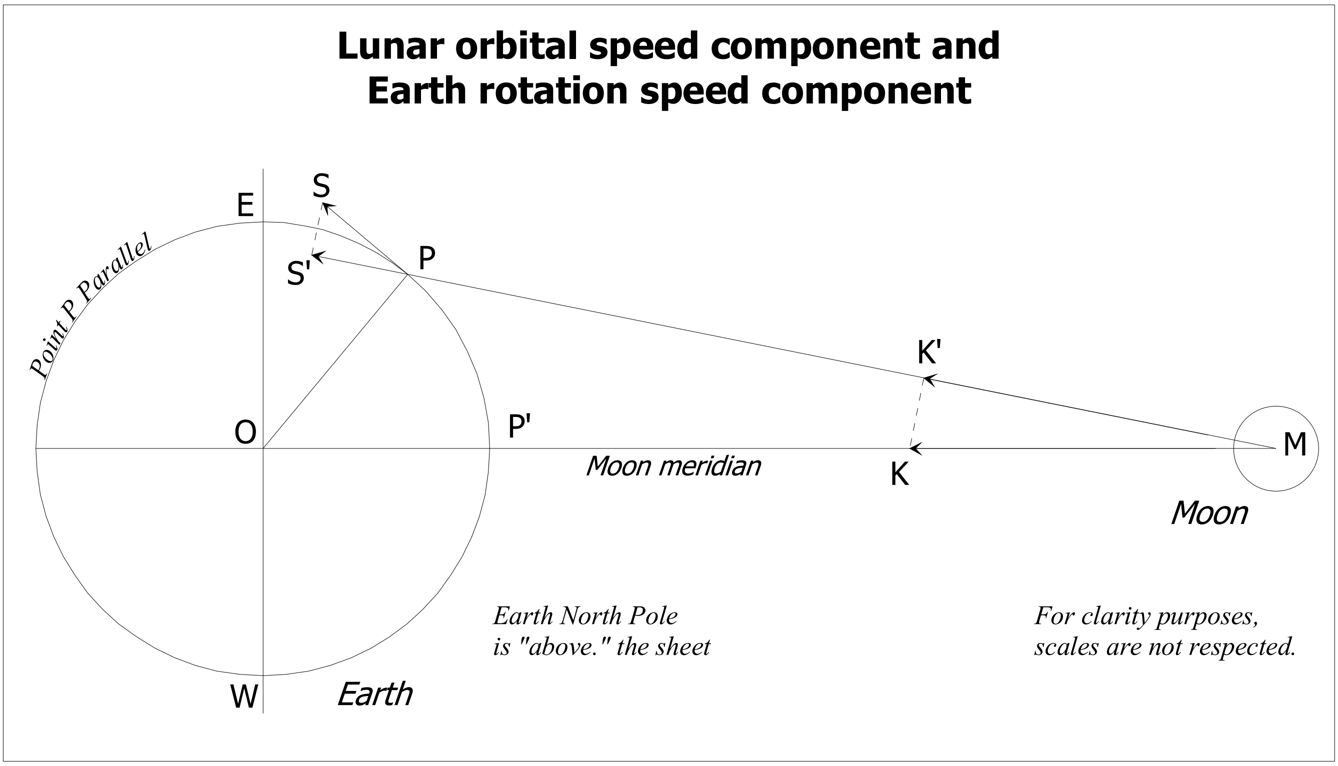


On the axis "Point P-Moon" (PM), vector MK' shows the radial speed component due to the motion of the Moon along the axis PM (consequence of the keplerian motion of the Moon along its orbit). Vector PS shows the tangential speed component at point P due to Earth's rotation. Vectors MK' and PS' show the projections on the axis "point P-Moon" (PM) of the components MK and PS respectively, both adding algebraically on the axis PM. Point P' is the image of point P in the Moon meridian. *(For clarity purposes, components MK, MK', PS and PS' are not to scale and highly magnified).*

At time of Moon transit at point P, points O (center of the Earth), P, P' and M are all contained in the meridian of point P. Under those conditions, point P and point P' are merged. As a result, the radial speed component PS' due to Earth rotation on the axis "Points P/P'-Moon" cancels out.

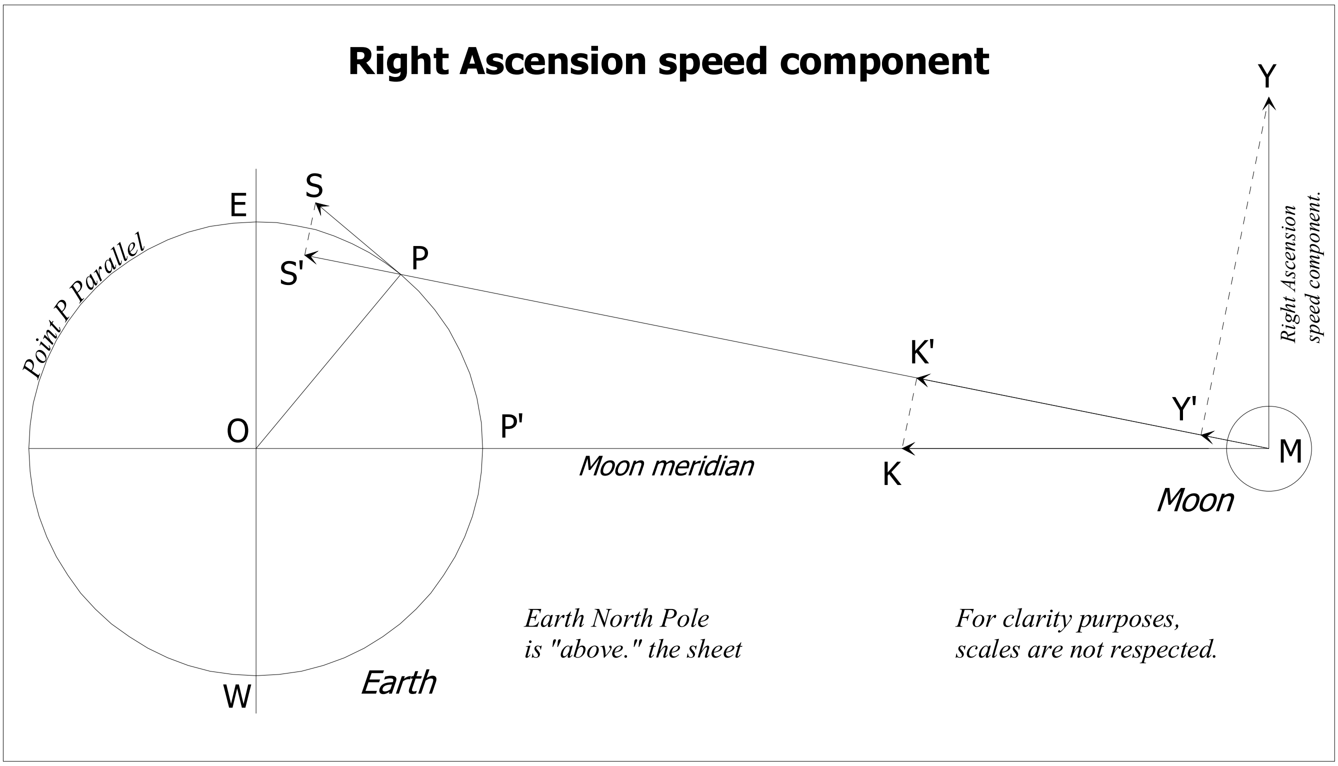
The component PS is maximum at the equator. The component PS' is maximum when the angle OPM equals 90°. At perigee or apogee Moon transit, both components MK and MK' cancel out. They are maximum when the Moon is at 90°, or 270°, true anomaly, between perigee and apogee.

In simple former algorithms, only those two components were taken into account to compute the resultant radial speed, hence the Doppler-Fizeau effect, at point P. Moreover, component PS' was directly added to component MK, just as if the axis PM was merged with the axis OM (see Part 1).

Orders of magnitude:

Speed component	Amplitude	Shift at 432 MHz	Shift at 10 GHz	Shift at 24 GHz
Tangential, equator	≈465 m/s	≈1340 Hz	≈32160 Hz	≈74450 Hz
Tangential, ±30° lat.	≈400 m/s	≈1155 Hz	≈27665 Hz	≈64050 Hz
Tangential, ±45° lat.	≈330 m/s	≈950 Hz	≈22745 Hz	≈52835 Hz
Tangential, ±60° lat.	≈230 m/s	≈665 Hz	≈15910 Hz	≈36825 Hz
Orbital, on axis OM	≈73 m/s	≈210 Hz	≈5050 Hz	≈11690 Hz

Note the high amount of frequency shift at and above 10 GHz.



On the axis "Point P-Moon" (PM), vector MK' shows the radial speed component due to the motion of the Moon along the axis PM, and vector MY, the speed component due to its motion in Right Ascension around the Earth. Vector PS shows the tangential speed component at point P due to Earth's rotation.

Vectors MK', MY' and PS' show the projections on the axis "point P-Moon" (PM) of the components MK, MY and PS respectively, all adding algebraically on the axis PM. Point P' is the image of point P in the Moon meridian. *(For clarity purposes, components MY, MY', MK, MK', PS and PS' are not to scale and highly magnified).*

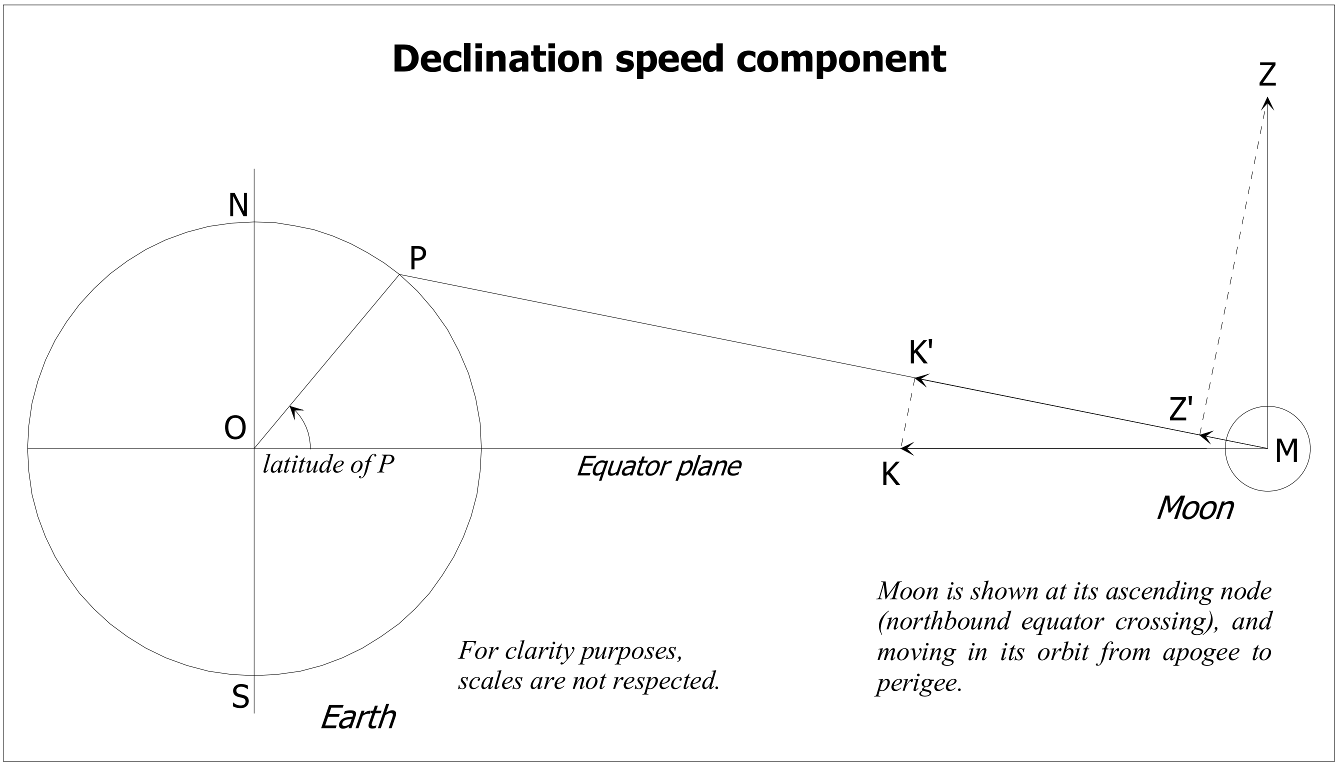
When Moon transits at point P, points O (center of the Earth), P, P' and M are all contained in the meridian of point P. Under those conditions, Point P and point P' are merged. As a result, both the radial speed component PS' due to Earth rotation on the axis "Points P/P'-Moon", and the component MY' due to the tangential speed component (Right Ascension motion), cancel out.

When P is not merged with P', the component MY' is no longer null and varies as a function of the angle MOP and as a function of Moon's position with respect to its perigee. The component MY' is maximum when angle OPM equals 90°, and when the Moon transits at perigee.

Orders of magnitude:

Speed component	Amplitude	Shift at 432 MHz	Shift at 10 GHz	Shift at 24 GHz
At equator	≈ 18 m/s	≈ 50 Hz	≈ 1245 Hz	≈ 2880 Hz
At ±30° latitude	≈ 16 m/s	≈ 45 Hz	≈ 1105 Hz	≈ 2560 Hz
At ±45° latitude	≈ 13 m/s	≈ 35 Hz	≈ 900 Hz	≈ 2080 Hz
At ±60° latitude	≈ 9 m/s	≈ 25 Hz	≈ 620 Hz	≈ 1440 Hz

As can be seen from this table, if it is legitimate to neglect the Right Ascension speed component at 432 MHz, its influence becomes quite significant at and above 10 GHz, and leads to even greater frequency shifts than those due to the primary speed components at 432 MHz.



On the axis "Point P-Moon" (PM), vector MK' shows the projection of the speed component MK due to Moon motion along the axis "Earth-Moon" (OM), and vector MZ', the projection of the component MZ due to Moon motion in Declination. *(For clarity purposes, components MK, MK', MZ and MZ' are highly magnified)*.

At time of Moon transit at point P, points O (center of the Earth), N, P and M are all contained in the same plane, the meridian of point P. As a result, the Earth rotation radial speed component (not visible here), on the axis "Point P-Moon" (PM), cancels out.

It then remains the Moon speed component MK on the axis "Earth-Moon" (OM), and the perpendicular component MZ, showing the Moon motion in Declination, their respective projections MK' and MZ' on the axis PM adding algebraically.

Component MK cancels out when Moon transits at perigee or apogee. Component MZ cancels out when Moon declination reaches its maximum (averaging ±27° in 2004). For latitudes between northernmost and southernmost declination, component MZ' cancels out when the Moon transits at true zenith above point P.

From mid-latitudes up to the poles, component MZ, and hence component MZ', are maximum at either ascending or descending node. The table gives component values when the Moon transits at either node.

Order of magnitude:

Speed component	Amplitude	Shift at 432 MHz	Shift at 10 GHz	Shift at 24 GHz
At ±40° latitude	≈ 5 m/s	≈ 14 Hz	≈ 350 Hz	≈ 800 Hz
At ±50° latitude	≈ 6 m/s	≈ 17 Hz	≈ 415 Hz	≈ 960 Hz
At ±70° latitude	≈ 7 m/s	≈ 20 Hz	≈ 485 Hz	≈ 1120 Hz

If the effect of component MZ' is actually negligible at 432 MHz, it becomes significant at and above 10 GHz. Note that if the Right Ascension speed component decreases with latitude, the Declination speed component increases with latitude.

Part 1: FORMER APPROXIMATE ALGORITHM

Before Personal Computers became widely available, it was already possible to compute the Doppler-Fizeau effect by means of "programmable pocket calculators". However, due to slow computing speed and reduced memory space, it was necessary to make significant simplifications, as follows:

1) Moon's radial speed.

It is computed, either numerically, or by derivating the Earth–Moon distance with respect to time, on the Earth–Moon axis. The Earth–Moon axis is not parallel to the Station–Moon axis. It can deviate by an amount of one degree at moonrise and moonset (parallax effect). To make things simpler, they are supposed to remain parallel. In other words, both axis are merged together.

2) Station speed relative to the Moon.

The station rotates together with the Earth. An acceptable approximation is to assume the speed component of this speed on the Earth–Moon axis mainly depends upon the latitude of the station, and upon the elevation and azimuth of the Moon relative to the station, for a given time *t*.

3) Resulting speed vr_{res}:

It is simply the sum of both speeds, according to the following formula:

$$vr_{\text{res}} = vr_{\text{L}} + 2\pi \, \text{n} \, \text{a}_0 \cdot \cos\varphi \cdot \cos El \cdot \sin Az$$

where:

vr_{L} : Moon's radial speed
n : Earth's spin rate
a_0 : Earth's equatorial radius
φ : Station's latitude
El : Moon's elevation at station
Az : Moon's azimuth at station

This leads to the Doppler-Fizeau effect Δf as observed on the echo at frequency f_0:

$$\Delta f \approx -2 f_0 \frac{vr_{\text{res}}}{\text{c} - vr_{\text{res}}}$$

where c is the speed of light.

Data given by these formulas are satisfying as far as lower bands (144 MHz, 432 MHz and to some extent, 1296 MHz) are concerned. But the higher the frequency increases, the more these computed data deviate from the observed values. Causes of these deviations can be summed up as follows:

1) Earth–Moon axis is merged with Station–Moon axis.

2) The parallax effect, which is responsible of the secondary Moon's radial speed components on the Station–Moon axis, themselves linked to apparent motions in Right Ascension and in Declination, is not taken into account. See above diagrams for full explanation of both components influence.

Part 2: DOPPLER-FIZEAU EFFECT ALGORITHM

This algorithm systematically makes use of rectangular coordinates and simple matrix processing.

1) EME Ground Station Rectangular Coordinates

Any "EME ground station" is accurately located on the Earth through its geographic coordinates (latitude φ, positive North, negative South, and longitude λ, positive East, negative West) and its altitude h expressed in the same unit as the equatorial Earth radius a_0 (see *Figure 1*).

The Earth can be taken as an ellipsoid of equatorial radius a_0 and flattening ε (see figure 1 and physical useful constants in appendix 5). The following formulas give the rectangular coordinates of the ground station (further on called "point P"):

$$\tan u = (1-\varepsilon)\tan\varphi \qquad \text{and} \qquad q = \frac{h}{a_0} \qquad \text{(1) and (2)}$$

point P associated column matrix P:

$$P = \begin{vmatrix} \cos u\cos\lambda + q\cos\varphi\cos\lambda \\ \cos u\sin\lambda + q\cos\varphi\sin\lambda \\ (1-\varepsilon)\sin u + q\sin\varphi \end{vmatrix} \qquad (3)$$

point P associated matrix mP:

$$mP = \begin{vmatrix} -\cos\lambda\sin\varphi & -\sin\lambda\sin\varphi & +\cos\varphi \\ -\sin\varphi & +\cos\lambda & 0 \\ +\cos\lambda\cos\varphi & +\sin\lambda\cos\varphi & +\sin\varphi \end{vmatrix} \qquad (4)$$

2) Moon Position

As for the ground station (point P), all rectangular coordinate sets are written using column matrix format, as follows (see Appendix 4 and *Figures 2 to 6*):

Le: column matrix of Moon's ecliptical coordinates.
La: column matrix of Moon's equatorial coordinates.
Lh: column matrix of Moon's hour coordinates.
Lp: column matrix of Moon's hour coordinates, shifted to point P, now new center of local hour coordinate frame.
Lz: column matrix of Moon's horizontal coordinates at point P.

These rectangular coordinates are all distances and are expressed in length units.
The column matrixes read as follows:

$$Le = \begin{vmatrix} Le_x \\ Le_y \\ Le_z \end{vmatrix} \quad La = \begin{vmatrix} La_x \\ La_y \\ La_z \end{vmatrix} \quad Lh = \begin{vmatrix} Lh_x \\ Lh_y \\ Lh_z \end{vmatrix} \quad Lp = \begin{vmatrix} Lp_x \\ Lp_y \\ Lp_z \end{vmatrix} \text{ and } Lz = \begin{vmatrix} Lz_x \\ Lz_y \\ Lz_z \end{vmatrix} \qquad \text{(5) to (9)}$$

Throughout the following, all directly time dependant data are supposed already computed and available at time t, as described further on:

-Earth's axis Obliquity Ω on the ecliptic.
-Greenwich Sidereal Angle G.
-Moon's ecliptic rectangular coordinates, as a column matrix, Le.

The following matrixes are then filled:

1) Obliquity associated matrix $m\Omega$:

$$m\Omega = \begin{vmatrix} 1 & 0 & 0 \\ 0 & +\cos\Omega & -\sin\Omega \\ 0 & +\sin\Omega & +\cos\Omega \end{vmatrix} \tag{10}$$

2) Greenwich Sidereal Angle associated matrix mG:

$$mG = \begin{vmatrix} +\cos G & +\sin G & 0 \\ -\sin G & +\cos G & 0 \\ 0 & 0 & 1 \end{vmatrix} \tag{11}$$

Chain computing immediately gives:

$$La = m\Omega \cdot Le \tag{12}$$
$$Lh = mG \cdot La \tag{13}$$
$$Lp = Lh - P \tag{14}$$
$$Lz = mP \cdot Lp \tag{15}$$

Hence Moon's Greenwich hour coordinates:

1) Greenwich Hour Angle γ and Declination δ (– sign indicates that Greenwich Hour Angle γ is counted up clockwise. See figure 4):

$$\gamma = \arctan\frac{-Lh_y}{Lh_x} \quad \text{and} \quad \delta = \arctan\frac{Lh_z}{\sqrt{Lh_x^2 + Lh_y^2}} \tag{16) and (17}$$

And Moon's local horizontal coordinates at point P (output data, for Moon tracking):

2) Azimuth Az and Elevation El (see figure 6):

$$Az = \arctan\frac{Lz_y}{Lz_x} \quad \text{and} \quad El = \arctan\frac{Lz_z}{\sqrt{Lz_x^2 + Lz_y^2}} \tag{18) and (19}$$

3) Distance Dp between point P and Moon's surface:

$$Dp = \sqrt{Lp_x^2 + Lp_y^2 + Lp_z^2} - \mathrm{a_L} \tag{20}$$

where $\mathrm{a_L}$ is Moon's radius (see appendix 5, physical useful constants).

3) Doppler-Fizeau Effect

Doppler-Fizeau Effect computing requires knowing at every instant t the speed of the Moon relative to the observation point P, on the axis "point P–Moon". This speed is called *radial speed*. It is computed using Bessel's method (see appendix 7 and *figure 7*).

In the case of Doppler-Fizeau Effect, Bessel's reference frame is defined as follows:

1) The Earth is set *fixed.*
2) The origin is the center O of the Earth.
3) The axis Ox is oriented positive Eastward.
4) The axis Oy is oriented positive Northward.
5) The axis Oz is parallel to the straight line joining point P to the Moon, positive sense from Earth center O towards the Moon.

This leads to (see *figure 7*):

1) The Earth's axis is always contained in plane O*yz*.
2) The reference frame rotates clockwise around the Earth's axis.
3) The Moon and point P both move in this frame.
4) Moon and point P abscissas and ordinates are identical *by definition*. Only heights are different.
5) At point P meridian transit, Moon center, point P itself and Earth axis are all three contained in plane O*yz*.

In order to obtain the speed components on the axis, derivatives of Moon's rectangular coordinates are first computed:

$\dot{L}e$: column matrix of Moon's ecliptical coordinates derivatives.
$\dot{L}a$: column matrix of Moon's equatorial coordinates derivatives.
$\dot{L}h$: column matrix of Moon's hour coordinates derivatives.
$\dot{L}p$: column matrix of Moon's hour coordinates derivatives, as shifted to point P.

These derivatives are expressed in length units per time units.

$$\dot{L}e = \begin{vmatrix} \dot{L}e_x \\ \dot{L}e_y \\ \dot{L}e_z \end{vmatrix} \qquad \dot{L}a = \begin{vmatrix} \dot{L}a_x \\ \dot{L}a_y \\ \dot{L}a_z \end{vmatrix} \qquad \dot{L}h = \begin{vmatrix} \dot{L}h_x \\ \dot{L}h_y \\ \dot{L}h_z \end{vmatrix} \qquad \dot{L}p = \begin{vmatrix} \dot{L}p_x \\ \dot{L}p_y \\ \dot{L}p_z \end{vmatrix} \qquad (21) \text{ to } (24)$$

Moon's ecliptical coordinates derivatives $\dot{L}e$ are supposed already computed and made available through the Tables of the Moon. Computing of other derivatives is then carried out in the various frames (speed components on the axis):

-Obliquity: it is taken as fixed over a short period. Hence derivatives are null.
-Point P: it is *fixed by definition* in the Hour reference frame. Hence derivatives are null.

Moon's equatorial coordinate derivatives can then be computed:

$$\dot{L}a = m\Omega \cdot \dot{L}e \qquad (25)$$

In order to compute Moon's hour coordinate derivatives, the Sidereal Angle G associated derivative matrix $m\dot{G}$ must first be filled:

$$m\dot{G} = 2\pi\mathrm{n} \cdot \begin{vmatrix} -\sin G & +\cos G & 0 \\ -\cos G & -\sin G & 0 \\ 0 & 0 & 0 \end{vmatrix} \qquad (26)$$

Where n is the Earth's spin rate, expressed in rotations (or cycles) per unit of time (see app. 5).
Hour coordinate derivatives then read as follows:

$$\dot{L}h = m\dot{G} \cdot La + mG \cdot \dot{L}a \qquad (27)$$

Point P being fixed in the Hour reference frame, we obtain:

$$\dot{P} = \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix} \quad \text{hence} \quad \dot{L}p = \begin{vmatrix} \dot{L}p_x \\ \dot{L}p_y \\ \dot{L}p_z \end{vmatrix} = \dot{L}h = \begin{vmatrix} \dot{L}h_x \\ \dot{L}h_y \\ \dot{L}h_z \end{vmatrix} \qquad (28) \text{ to } (30)$$

In the Greenwich Hour frame, the Hour Angle γ_B and the Declination δ_B of Bessel's axis O*z* are defined at any time *t*. These angles are identical to the Local Hour Angle and Local Declination of the Moon, in the local frame shifted at point P, but are taken from the Earth's center O (see appendix 4 and *figure 5*, appendix 7 and *figure 7*), hence:

$$\gamma_B = \arctan\frac{-Lp_y}{Lp_x} \quad \text{and} \quad \delta_B = \arctan\frac{Lp_z}{\sqrt{Lp_x^2 + Lp_y^2}} \qquad \text{(31) and (32)}$$

The associated matrix *mB* can then be filled:

$$mB = \begin{vmatrix} +\sin\gamma_B & +\cos\gamma_B & 0 \\ -\cos\gamma_B \sin\delta_B & +\sin\gamma_B \sin\delta_B & +\cos\delta_B \\ +\cos\gamma_B \cos\delta_B & -\sin\gamma_B \cos\delta_B & +\sin\delta_B \end{vmatrix} \qquad (33)$$

Hence the coordinates of Moon and point P in Bessel's reference frame:

$$Lb = mB \cdot Lh \quad \text{and} \quad Pb = mB \cdot P \qquad \text{(34) and (35)}$$

Bessel's axis coordinates derivatives:

Derivatives of Bessel's axis Hour Angle, of its Sine and of its Cosine:

$$\dot{\gamma}_B = \frac{\dot{L}p_y Lp_x - Lp_y \dot{L}p_x}{Lp_x^2 + Lp_y^2} \qquad (36)$$

$$\dot{\sin}\,\gamma_B = +\dot{\gamma}_B \cos\gamma_B \quad \text{and} \quad \dot{\cos}\,\gamma_B = -\dot{\gamma}_B \sin\gamma_B \qquad \text{(37) and (38)}$$

Derivatives of Bessel's axis Declination, of its Sine and of its Cosine:

$$\dot{\delta}_B = \frac{\dot{L}p_z(Lp_x^2 + Lp_y^2) - Lp_z(Lp_x\dot{L}p_x + Lp_y\dot{L}p_y)}{(Lp_x^2 + Lp_y^2 + Lp_z^2)\sqrt{Lp_x^2 + Lp_y^2}} \qquad (39)$$

$$\dot{\sin}\,\delta_B = +\dot{\delta}_B \cos\delta_B \quad \text{and} \quad \dot{\cos}\,\delta_B = -\dot{\delta}_B \sin\delta_B \qquad \text{(40) and (41)}$$

Bessel's axis associated matrix derivative $m\dot{B}$:

$$m\dot{B} = \begin{vmatrix} +\dot{\gamma}_B \cos\gamma_B & -\dot{\gamma}_B \sin\gamma_B & 0 \\ -\dot{\gamma}_B \sin\gamma_B \sin\delta_B - \dot{\delta}_B \cos\delta_B \cos\gamma_B & -\dot{\gamma}_B \cos\gamma_B \sin\delta_B + \dot{\delta}_B \cos\delta_B \sin\gamma_B & -\dot{\delta}_B \sin\delta_B \\ +\dot{\gamma}_B \sin\gamma_B \cos\delta_B - \dot{\delta}_B \sin\delta_B \cos\gamma_B & +\dot{\gamma}_B \cos\gamma_B \cos\delta_B + \dot{\delta}_B \sin\delta_B \sin\gamma_B & +\dot{\delta}_B \cos\delta_B \end{vmatrix} \qquad (42)$$

Note: all angular derivatives are expressed in radians per unit of time.

Derivatives of Moon's motion and point P motion in Bessel's frame can then be computed:

$$\dot{L}b_{\mathrm{P}} = m\dot{B} \cdot Lh + mB \cdot \dot{L}h \quad \text{and} \quad \dot{P}b = m\dot{B} \cdot P \qquad \text{(43) and (44)}$$

Moon's Radial Speed vr_{P} as observed from point P is measured on the axis O*z* of Bessel's frame. The same computing can be done likewise for a second point Q [see (50)]. In the Bessel's reference frame associated to the system "point Q–Moon", Moon's derivative coordinates $\dot{L}b_{\mathrm{Q}}$, and point Q derivative coordinates $\dot{Q}b$ can be computed.

This finally leads to:

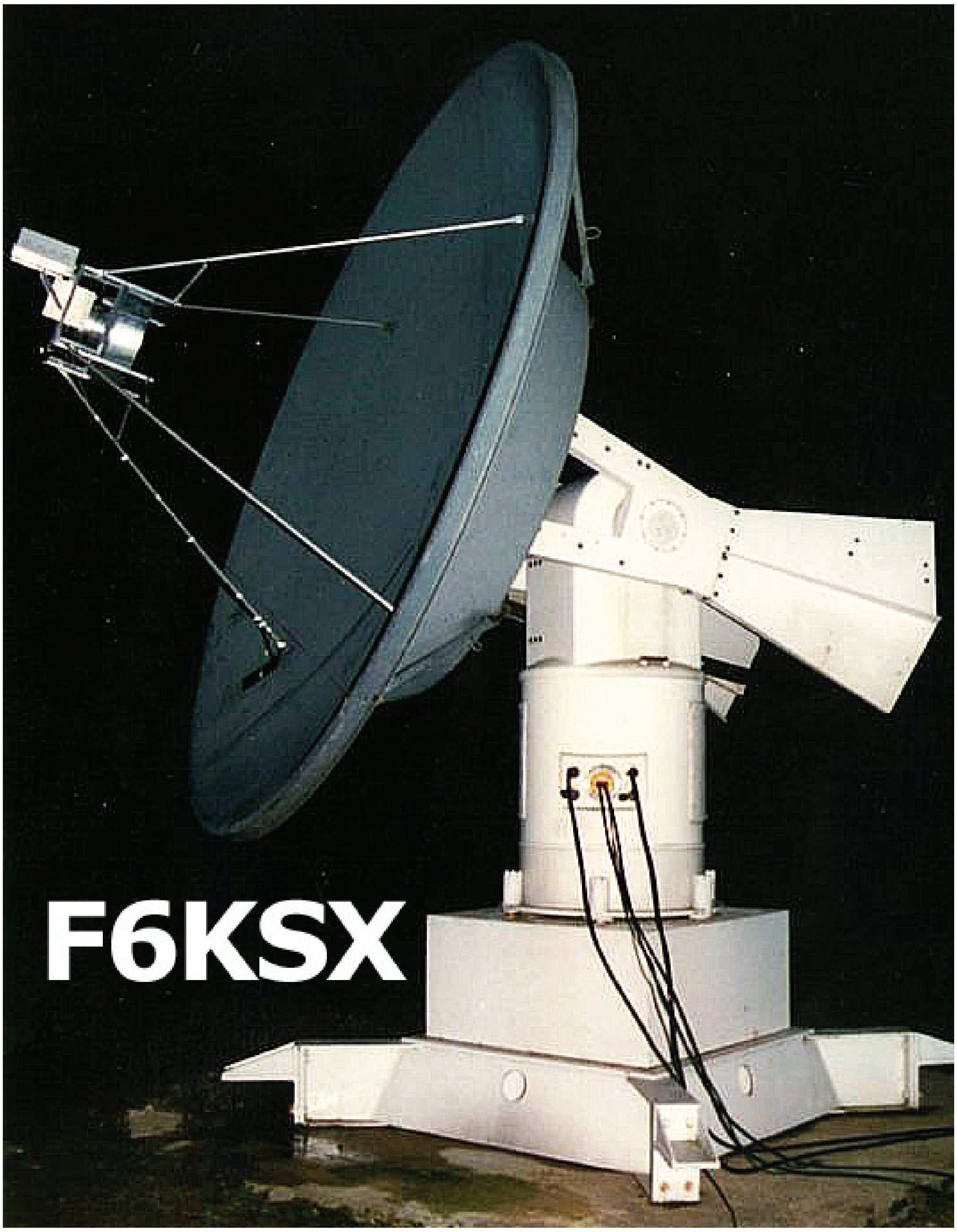
$$\boxed{vr_{\mathrm{P}} = \dot{L}b_{\mathrm{P}z} - \dot{P}b_z} \quad \text{and} \quad \boxed{vr_{\mathrm{Q}} = \dot{L}b_{\mathrm{Q}z} - \dot{Q}b_z} \qquad \text{(45) and (46)}$$

This gives the Doppler-Fizeau effects Δf_{P} and Δf_{Q} on the EME echo at the frequency f_0, as observed respectively by station P and station Q:

$$\boxed{\Delta f_{\mathrm{P}} \approx -2 f_0 \frac{vr_{\mathrm{P}}}{\mathrm{c} - vr_{\mathrm{P}}}} \quad \text{and} \quad \boxed{\Delta f_{\mathrm{Q}} \approx -2 f_0 \frac{vr_{\mathrm{Q}}}{\mathrm{c} - vr_{\mathrm{Q}}}} \qquad \text{(47) and (48)}$$

This also finally gives the mutual effect as observed on signals sent by P and received by Q, and vice-versa:

$$\boxed{\Delta f_{\mathrm{PQ}} = \Delta f_{\mathrm{QP}} \approx - f_0 \left[\frac{vr_{\mathrm{P}}}{\mathrm{c} - vr_{\mathrm{P}}} + \frac{vr_{\mathrm{Q}}}{\mathrm{c} - vr_{\mathrm{Q}}} \right]} \qquad (49)$$



Part 3: POLARIZATION EFFECT

This is the orientation difference between the polarization plane of both stations P and Q, as a function of: 1) their respective geographical position, 2) their respective position relative to the Moon. This parameter can be useful for stations using El-Az mounted antennas with fixed linear polarization, either horizontal or vertical, especially on the microwave bands.

Point Q rectangular coordinates are obtained in the same way as for point P. u_Q and q_Q are first computed using formulas (1) and (2), and the column matrix Q is then filled:

$$Q = \begin{vmatrix} \cos u_Q \cos \lambda_Q + q_Q \cos \varphi_Q \cos \lambda_Q \\ \cos u_Q \sin \lambda_Q + q_Q \cos \varphi_Q \sin \lambda_Q \\ (1 - \varepsilon) \sin u_Q + q_Q \sin \varphi_Q \end{vmatrix} \qquad (50)$$

Bessel's method is also used here (see appendix 7 and *figure 8*). In this case, the axis O*z* is merged to the axis Earth–Moon, and respective positions of point P and point Q are computed in this frame.

The associated matrix *mL* is then filled:

$$mL = \begin{vmatrix} +\sin\gamma & +\cos\gamma & 0 \\ -\cos\gamma \sin\delta & +\sin\gamma \sin\delta & +\cos\delta \\ +\cos\gamma\cos\delta & -\sin\gamma\cos\delta & +\sin\delta \end{vmatrix} \qquad (51)$$

Where γ and δ are the Greenwich Hour Angle and the Declination of the Moon, for a given time *t*. Coordinates Pl and Ql of points P and Q in this frame now read:

$$Pl = mL \cdot P \quad \text{and} \quad Ql = mL \cdot Q \qquad (52) \text{ and } (53)$$

This leads to the "polarization angle" Pol_{PQ} between directions OP' and OQ' (see figure 8):

$$Pol_{PQ} = \arctan \left| \frac{Pl_y Ql_x - Pl_x Ql_y}{Pl_x Ql_x + Pl_y Ql_y} \right| \qquad (54)$$

Appendix 1: Reference Frames and Coordinates

-Rectangular coordinates are also called cartesian coordinates.
-Spherical coordinates are also called polar coordinates.
-Horizontal coordinates are also called azimuthal coordinates.

The Earth is supposed to be *fixed*. EME stations (points P and Q) being bound to Earth, are also *fixed*, their respective position being given by their geographical coordinates and their distance to the center of the Earth. Hence, *apparent motions* around the Earth are dealt with.

As a consequence, the Ecliptical, Equatorial and Greenwich Hour reference frames, whose origin is the center of the Earth, are fixed.

Likewise, the Local Hour frame and the Local Horizontal frame, both linked to the EME station, are also *fixed.*

On the other hand, Bessel's frames are instantaneous mobile frames, also called dynamic reference frames. Their goal is to simplify computing (see appendix 3 and 7).

Appendix 2: Getting from one Reference Frame into the other

Two kinds of frame changes are defined: rotation and shift, combinations of both being possible.

Rotation

It proceeds around one of the frame axis, either counter-clockwise (Earth's spin sense), or clockwise (reverse Earth's spin sense). The column matrix of the point in the new frame is the product of a matrix whose coefficients are trigonometric functions of the rotation angle, and of a column matrix whose coefficients are the rectangular coordinates of the point in the previous frame.

Shift

This is a mere origin shift. Axis directions of the new frame remain parallel to that of the previous one. To get from one frame to the other, the new frame origin coordinates in the previous frame are subtracted to the coordinates of the point in the previous frame.

Practical algorithmic processing

Getting from the Ecliptic frame into the Equatorial frame is carried out through a single clockwise rotation of angle Ω (Earth axis obliquity on the ecliptic) around the Ox axis (spring equinox axis, or vernal point axis). Ox axis is hence common to both frames.

Getting from the Equatorial frame into the Greenwich Hour frame is carried out through a single counter-clockwise rotation of angle G (Greenwich Sidereal Angle) around the Oz axis (Earth's polar axis). In that frame, the Greenwich Hour Angle γ is counted up clockwise, or positive westwards.

Getting from the Greenwich Hour frame into the Local Hour frame is carried out through a single origin shift from one frame into the other, the three axis of both frames remaining parallel to each other. Like the Greenwich Hour Angle, the Local Hour Angle is counted up clockwise. Note: this Local Hour frame should rather be called "Shifted Local Hour frame". The "True Local Hour frame" still requires a final rotation of longitude λ (longitude of point P), in order to obtain the correct Local Hour coordinates at point P, if the antennas at station P are installed on a polar mount. For further computing, this step is not necessary and is therefore skipped.

Getting from the Local Hour frame into the Local Horizontal frame (for El-Az mounted antennas) is carried out this time through a double rotation: a first counter-clockwise rotation of angle $\lambda + 180°$ (point P longitude, increased by a half-turn) around the axis Pz, and a second counter-clockwise rotation of angle $90° - \varphi$ (point P colatitude) around the axis Py. The new frame is such that the azimuth origin is North, and the azimuth angle is counted up clockwise.

Likewise, getting from fixed frames into dynamic mobile Bessel's frames is carried out through double rotations. The coefficients of the frame associated matrix are combined trigonometric functions of both rotation angles (Declination and Local Hour angle of Bessel's axis). Because Bessel's frames and their associated matrixes are instantaneous, they are computed for each time *t*.

Appendix 3: Advantages of Bessel's Method

The spherical trigonometry computing method separately deals with the four components of the radial speed, and then, recombines the separate results to obtain the final speed. This method generally leads to bulky computing code, with the risk of editing errors later difficult to trace.

On the other hand, Bessel's method implicitly and globally takes into account all factors involved in the resulting radial speed of the Moon and processes them all at once. This method, associated with rectangular coordinates and matrix processing, leads to rather simple computing code, greatly limiting the risk of editing errors.

Appendix 4: Fixed Reference Frames

Figure 1: Point P (EME station, observer) rectangular coordinates. The origin is the center O of the Earth. The Earth's flattening and altitude of point P are taken into account in computing rectangular coordinates.

Figure 2: Ecliptical coordinates. The origin is the center O of the Earth. The reference plane is the orbital plane of the Earth (also called "Ecliptic"). Moon's position is computed in this frame in function of time (Oables of the Moon). The axis O*z* is perpendicular to the ecliptic.

Figure 3: Equatorial coordinates. The origin is the center O of the Earth. The reference plane is the Earth's equator plane. Its inclination on the ecliptic amounts to about 23.44°. This inclination is called "polar axis obliquity". The axis O*x*, common to both frames, is the intersection of the Equator with the Ecliptic. The axis O*z* is the spin axis of the Earth, oriented from South to North.

Figure 4: Greenwich Hour coordinates. The origin is the center O of the Earth. The reference plane is the Earth's Equator plane. The axis O*x* is the intersection of the Equator with the Greenwich meridian. The axis O*z* is the spin axis of the Earth, oriented from South to North. Greenwich Hour Angle is counted up clockwise.

Figure 5: Local Hour coordinates. The origin is the observation point P. The reference plane is the plane parallel to the equator and containing point P. The axis P*x* is parallel to the axis O*x*, the axis P*y* is parallel to the axis O*y* and the axis P*z* is parallel to the Earth axis O*z*, which remains oriented from South to North. Local Hour Angle is counted up clockwise.

Figure 6: Local Horizontal (or Azimuthal) coordinates. The origin is the observation point P. The reference plane is the horizontal plane containing point P. The axis P*x* is oriented towards North, the axis P*y* towards East and the axis P*z* towards Zenith. Azimuth is counted up clockwise from North.

Appendix 5: Physical useful constants (IERS 1992)

IERS = International Earth Rotation Service.
Home page: http://hpiers.obspm.fr/
Physical useful constant direct access: http://hpiers.obspm.fr/eop-pc/models/constants.html

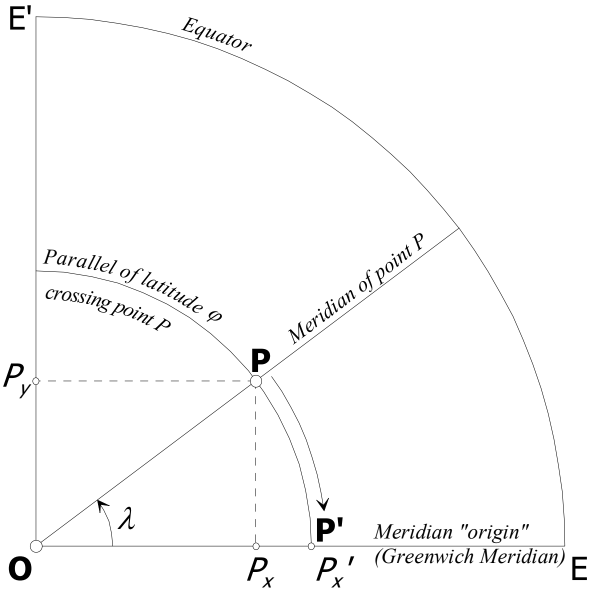
Constant	Symbol	Value	Unit	Other Format
Earth's equatorial radius	a_0	$6.3781363 \cdot 10^6$	meter	6378.1363 km
Earth's flattening	ε	0.003352892	(dimensionless)	1/298.257
Earth's spin rate	n	1.0027379093508	round / day	11.6057628397 µHz
Earth's polar axis Obliquity	Ω	23.439281083	degree	23°26'21".4119
Moon's radius	a_L	$1.73660068 \cdot 10^6$	meter	$0.272274 \cdot a_0$
Speed of Light	c	$2.99792458 \cdot 10^8$	meter / second	299792.458 km/s

Appendix 6: Short historical note

In the acoustic wave domain, the Austrian physicist Christian Doppler (1803–1853) discovered and described in 1842 the phenomenon which consists in the apparent variation of the frequency of a continuous wave source, when this source moves relatively to an observer, or vice-versa. In 1849, the French physicist Hippolyte Fizeau (1819–1896) made the same observations with light wave sources (light waves were later recognized as part of the electromagnetic wave spectrum) and extended to light the results obtained by Doppler in the acoustic wave domain, and hence demonstrated the identity of both phenomena. The unique phenomenon was later designated after their names, and is today known as "Doppler-Fizeau effect", often simply shorted as "Doppler".

Figure 1

Rectangular coordinates of point P on the Earth ellipsoid



The point P is defined through its latitude φ, its longitude λ and its altitude q = P'Q.

The point P' is the image of point P in the Greenwich "origin" meridian plane EON, obtained through rotation of longitude λ around the polar axis ON.

OE is the Earth's equatorial radius, taken as unity. OF = OE.

The latitude and altitude of point P' are equal to that of point P.

Q is the intersection point of the vertical at P' on the ellipsoid quarter EON. G is the intersection point of this same vertical with the equator. The angle HGP' is the geographic latitude of P', hence that of P.

GK is parallel to OM.

This leads to the following relationships:

Ellipsoid flattening ε:

$$\varepsilon = \frac{\text{OF - ON}}{\text{OF}} = 1 - \frac{\text{ON}}{\text{OF}}$$

Hence:

$$\frac{\text{ON}}{\text{OF}} = \frac{\text{HQ}}{\text{HM}} = \frac{\text{HK}}{\text{HQ}} = 1 - \varepsilon$$

$$\text{HK} = (1-\varepsilon)\text{HQ}$$

And:

$$\tan u = \frac{\text{HM}}{\text{HO}} = \frac{\text{HK}}{\text{HG}} = (1-\varepsilon)\frac{\text{HQ}}{\text{HG}}$$

$$\tan\varphi = \frac{\text{HQ}}{\text{HG}}$$

This leads to:

$$\tan u = (1-\varepsilon)\tan\varphi$$

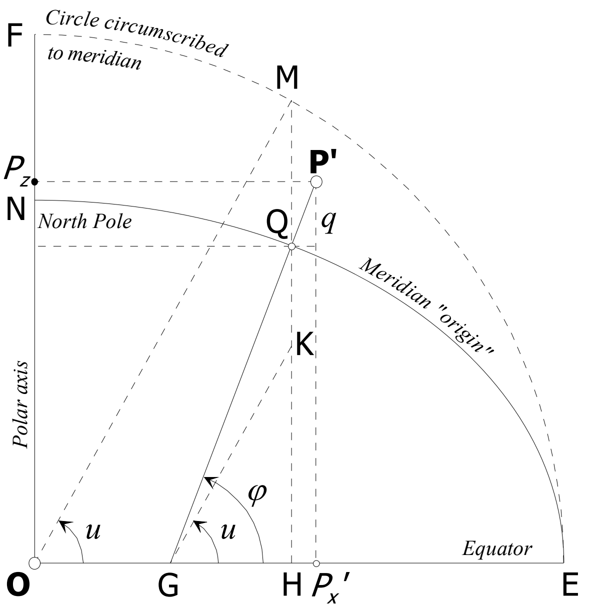
Thence the coordinates P_x' and P_z

$$P_x' = \cos u + q\cos\varphi$$

$$P_z = (1-\varepsilon)\sin u + q\sin\varphi$$

And finally the coordinates P_x , P_y and P_z

$$P_x = \cos u\cos\lambda + q\cos\varphi\cos\lambda$$
$$P_y = \cos u\sin\lambda + q\cos\varphi\sin\lambda$$
$$P_z = (1-\varepsilon)\sin u + q\sin\varphi$$



For sketch clarity, the ellipsoid flattening ε and the altitude q of point P' have been greatly magnified.

The equatorial radius OE = a_0 is taken here as unity.

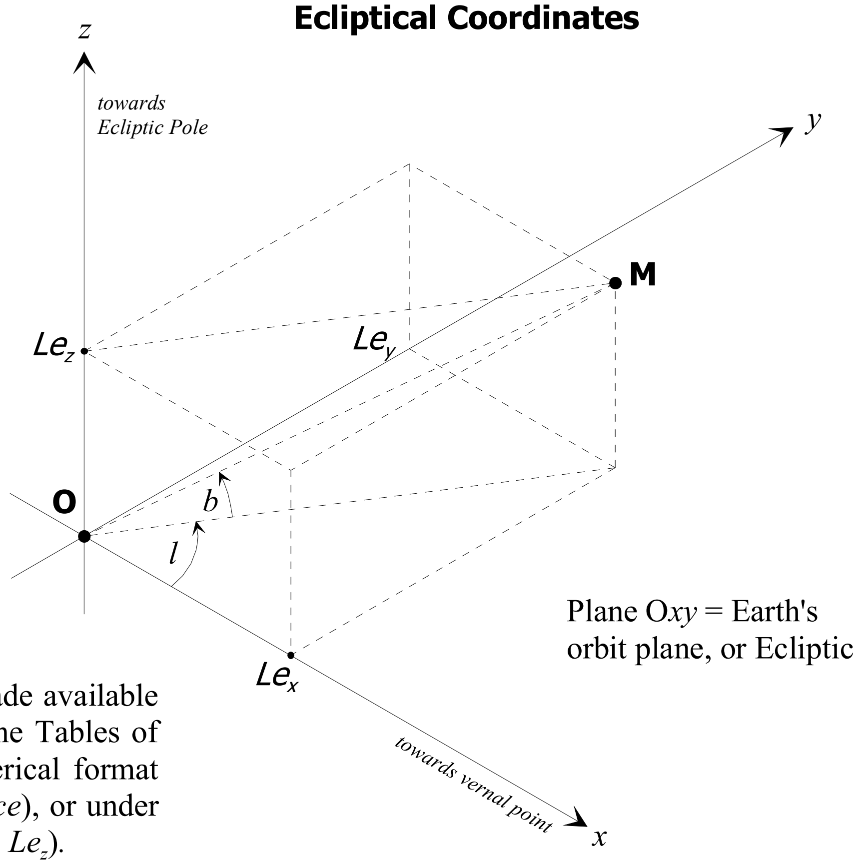
Figure 2

Ecliptical Coordinates

$b =$ ecliptical latitude

$l =$ ecliptical longitude

$\mathrm{OL} =$ Earth – Moon distance



$Le_x = \mathrm{OL} \cos l \cos b$

$Le_y = \mathrm{OL} \sin l \cos b$

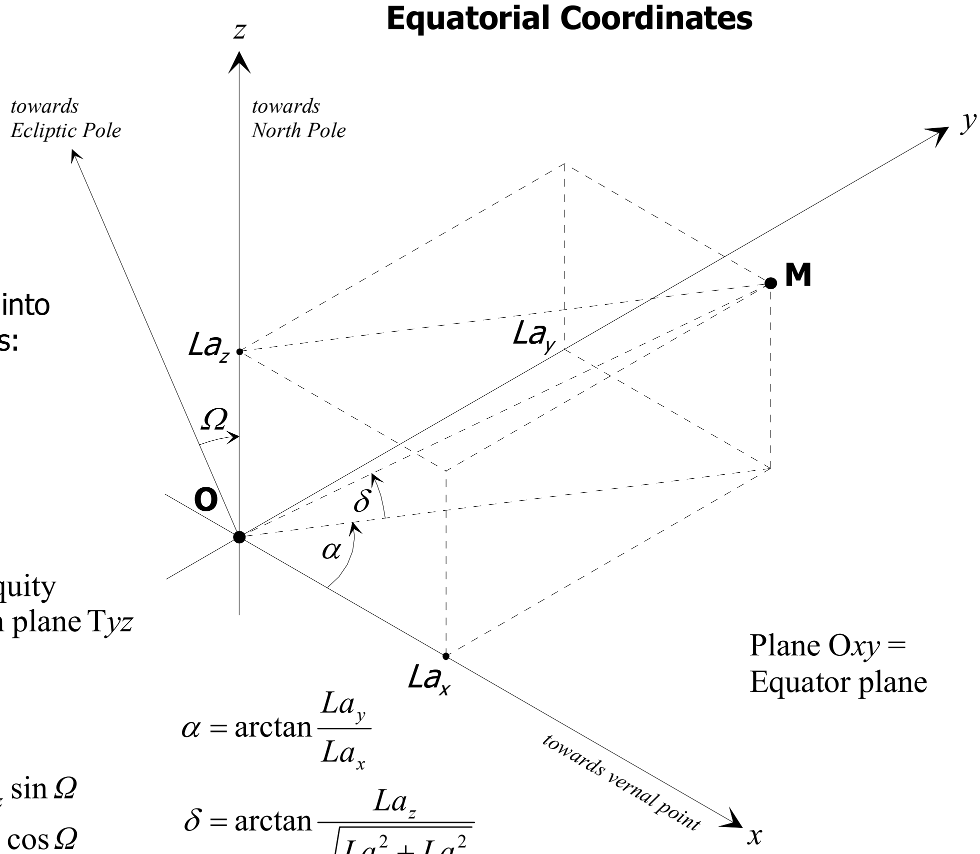
$Le_z = \mathrm{OL} \sin b$

Ecliptical coordinates are made available for every instant *t* through the Tables of the Moon, either under spherical format (*Longitude, Latitude, Distance*), or under rectangular format (Le_x , Le_y , Le_z).

Figure 3

Equatorial Coordinates

Getting from ecliptical coordinates into equatorial coordinates:



$\delta =$ Declination

$\alpha =$ Right Ascension

$\Omega =$ Earth's axis Obliquity on the Ecliptic, in plane Tyz

$La_x = Le_x$

$La_y = Le_y \cos \Omega - Le_z \sin \Omega$

$La_z = Le_y \sin \Omega + Le_z \cos \Omega$

$$\alpha = \arctan \frac{La_y}{La_x}$$

$$\delta = \arctan \frac{La_z}{\sqrt{La_x^2 + La_y^2}}$$

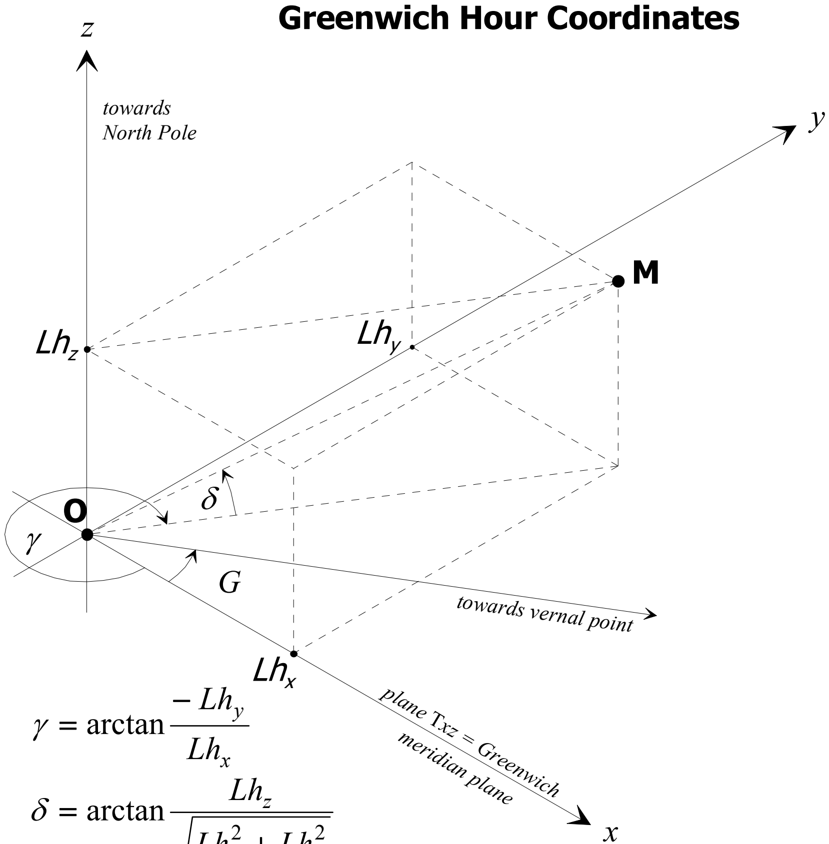
Figure 4

Greenwich Hour Coordinates

Getting from
Equatorial coordinates into
Greenwich Hour coordinates:

δ = Declination
γ = Greenwich Hour Angle
G = Greenwich Sidereal Angle

Greenwich Hour Angle γ is counted up clockwise.



$$Lh_x = +La_x \cos G + La_y \sin G$$
$$Lh_y = -La_x \sin G + La_y \cos G$$
$$Lh_z = +La_z$$

$$\gamma = \arctan \frac{-Lh_y}{Lh_x}$$
$$\delta = \arctan \frac{Lh_z}{\sqrt{Lh_x^2 + Lh_y^2}}$$

Figure 5

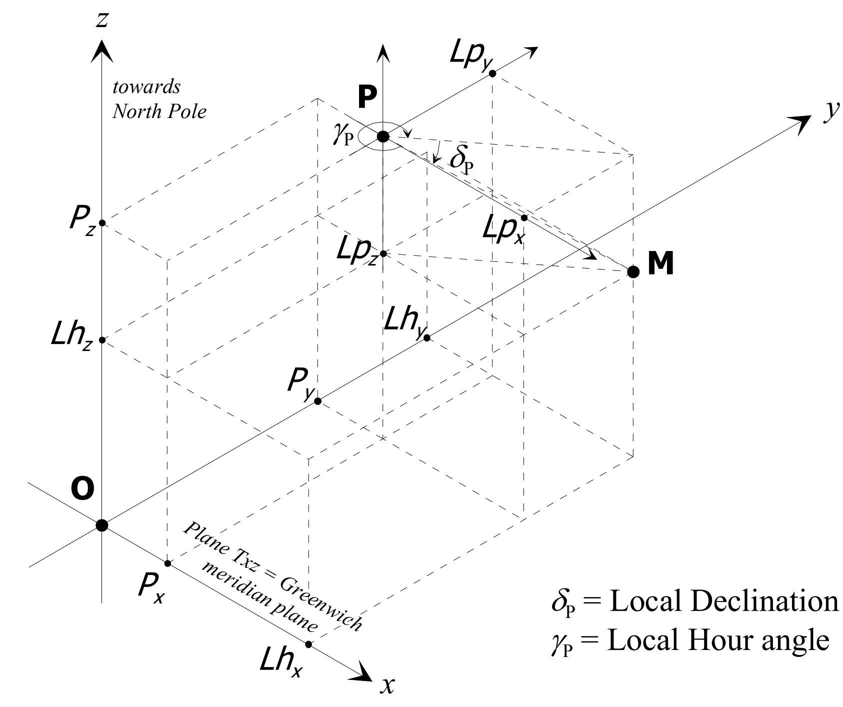
Local Hour Coordinates

Getting from
Greenwich Hour Coordinates
into Local Hour Coordinates:

$$Lp_x = Lh_x - P_x$$
$$Lp_y = Lh_y - P_y$$
$$Lp_z = Lh_z - P_z$$

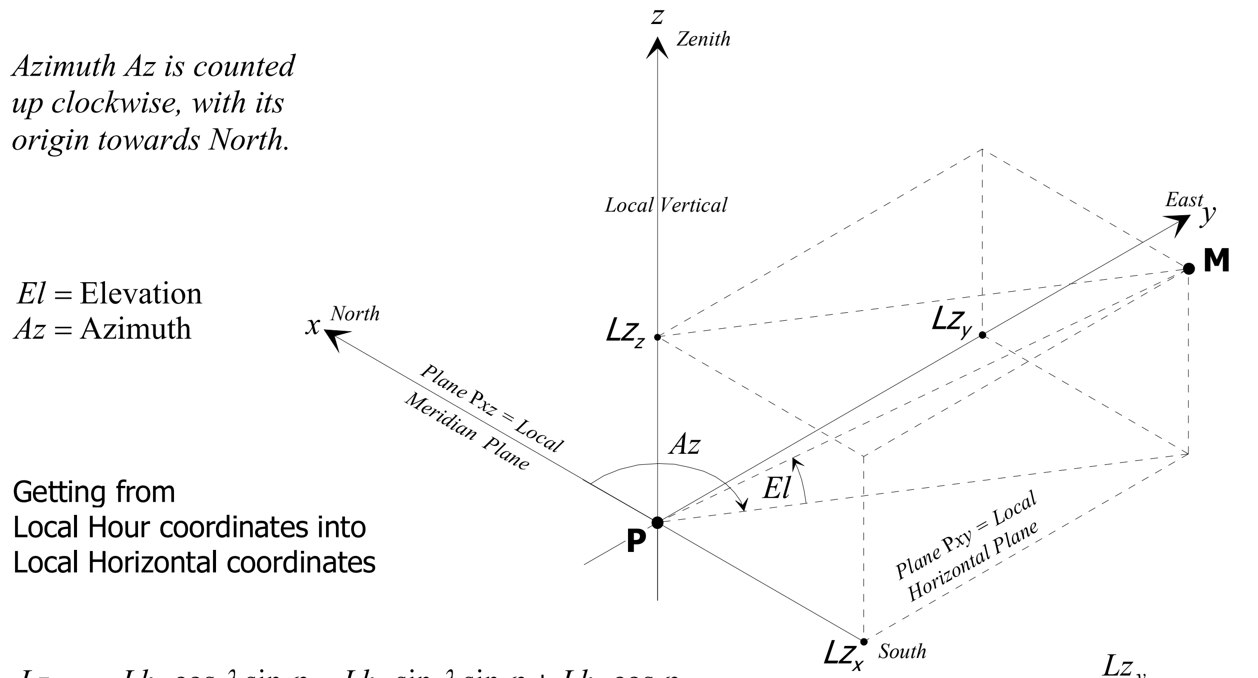
$$\gamma_P = \arctan \frac{-Lp_y}{Lp_x}$$
$$\delta_P = \arctan \frac{Lp_z}{\sqrt{Lp_x^2 + Lp_y^2}}$$

Local Hour Angle γ_P is counted up clockwise.



δ_P = Local Declination
γ_P = Local Hour angle

Figure 6



$$Lz_x = -Lh_x \cos\lambda \sin\varphi - Lh_y \sin\lambda \sin\varphi + Lh_z \cos\varphi$$
$$Lz_y = -Lh_x \sin\varphi + Lh_y \cos\lambda$$
$$Lz_z = +Lh_x \cos\lambda \cos\varphi + Lh_y \sin\lambda \cos\varphi + Lh_z \sin\varphi$$

$$Az = \arctan \frac{Lz_y}{Lz_x}$$
$$El = \arctan \frac{Lz_z}{\sqrt{Lz_x^2 + Lz_y^2}}$$

Appendix 7: Bessel's Reference Frames

The famous German astronomer and mathematician Friedrich Wilhelm Bessel (1784 – 1846) developed the *instantaneous dynamic reference frame method* (often referred to as "Bessel's method") in order to simplify Sun and Moon eclipse computing. This method deals with the *relative apparent motions*. First, *true motions* are computed, and then, their mutual combinations in order to obtain the *relative motions* as seen by the observer (here, the EME station located at point P). Any Bessel's reference frame is defined by the Greenwich Hour Angle and the Declination of its axis O*z*, and by its reference plane O*xy*, or Bessel's fundamental plane, containing the origin O.

Figure 7: Doppler-Fizeau effect. For a given instant *t,* the diagram shows the reference frame itself, with the (not to scale for clarity) position of the Moon and point P in this frame. The Earth being supposed *fixed*, the frame, centered on the origin O, turns around the Earth, according to the *relative apparent motion* of the Moon *around* point P. As a result, both the Moon and point P continuously move in this frame. In the case of the Doppler-Fizeau effect, only the relative motions of the Moon and point P on the straight line joining the center of the Moon to point P are taken into account, this axis being parallel to the frame O*z* axis. Computing of the Moon's rectangular coordinate derivatives and their further matrix processing along O*z* axis give the speed components of both the Moon and point P on this axis, and hence finally give the speed of the Moon relative to point P. This is the *point P to Moon radial speed*, from which the Doppler-Fizeau effect, as observed from point P, is computed.

Figure 8: Polarization effect. For a given instant *t,* the diagram shows the reference frame itself, the position of point P and that of point Q in the frame, the axis O*z* being this time merged to the straight line joining the Earth's center to Moon's center. The "polarization" angle is the angle between both planes PP'O and QQ'O, these planes crossing each other along the axis O*z*.

Figure 7

Doppler-Fizeau Effect
(Bessel's method)

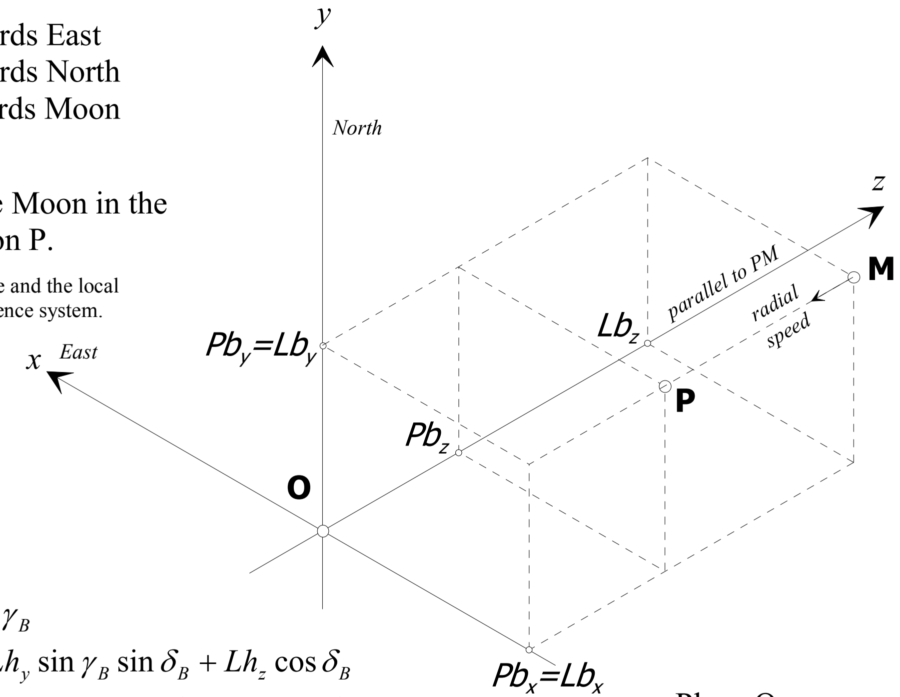
Bessel's reference system:
Ox axis: positive sense towards East
Oy axis: positive sense towards North
Oz axis: positive sense towards Moon and parallel to PM

Lp are the coordinates of the Moon in the local hour system centered on P.

γ_B et δ_B are identical to the local hour angle and the local declination of the Moon, in the local reference system.

$$\gamma_B = \arctan \frac{-Lp_y}{Lp_x}$$

$$\delta_B = \arctan \frac{Lp_z}{\sqrt{Lp_x^2 + Lp_y^2}}$$



$$Lb_x = +Lh_x \sin\gamma_B + Lh_y \cos\gamma_B$$

$$Lb_y = -Lh_x \cos\gamma_B \sin\delta_B + Lh_y \sin\gamma_B \sin\delta_B + Lh_z \cos\delta_B$$

$$Lb_z = +Lh_x \cos\gamma_B \cos\delta_B - Lh_y \sin\gamma_B \cos\delta_B + Lh_z \sin\delta_B$$

$$Pb_z = +P_x \cos\gamma_B \cos\delta_B + P_y \sin\gamma_B \cos\delta_B + P_z \sin\delta_B$$

Plane Oxy = fundamental Bessel's plane

Figure 8

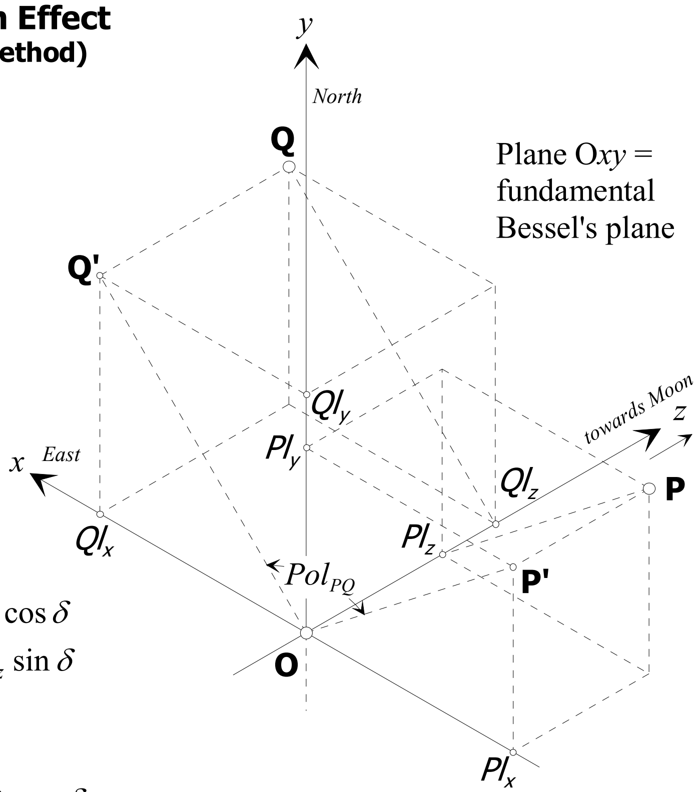
Polarization Effect
(Bessel's method)

Bessel's reference system:
Ox axis: positive sense towards East
Oy axis: positive sense towards North
Oz axis: Earth-Moon axis

Plane Oxy = fundamental Bessel's plane

$$\gamma = \arctan \frac{-Lh_y}{Lh_x}$$

$$\delta = \arctan \frac{Lh_z}{\sqrt{Lh_x^2 + Lh_y^2}}$$



$$Pl_x = +P_x \sin\gamma + P_y \cos\gamma$$

$$Pl_y = -P_x \cos\gamma \sin\delta + P_y \sin\gamma \sin\delta + P_z \cos\delta$$

$$Pl_z = +P_x \cos\gamma \cos\delta - P_y \sin\gamma \cos\delta + P_z \sin\delta$$

$$Ql_x = +Q_x \sin\gamma + Q_y \cos\gamma$$

$$Ql_y = -Q_x \cos\gamma \sin\delta + Q_y \sin\gamma \sin\delta + Q_z \cos\delta$$

$$Ql_z = +Q_x \cos\gamma \cos\delta - Q_y \sin\gamma \cos\delta + Q_z \sin\delta$$

$$Pol_{PQ} = \arctan \left| \frac{Pl_y Ql_x - Pl_x Ql_y}{Pl_x Ql_x + Pl_y Ql_y} \right|$$

Appendix 8: Some typical Doppler-Fizeau effect Curves

These curves show the various aspects of the Doppler-Fizeau effect on the frequency of the Moon echo at 10 GHz, for different positions of the Moon along its orbit (declination and radial speed). *Remember:* at 10368 MHz, a speed of ±*1 m/s* generates a frequency shift of ±*69 Hz* on the echo.

Figure 9: Here, the Moon is close to its southernmost declination and on its way towards the apogee almost at "fu1l speed". In that case, the frequency down shift is practically linear, from moonrise to moonset. Note that the shift is null about one hour before moon transit. At instant of transit, the right ascension speed component, as well as that due to the earth rotation, are both null. The declination being almost stationary, its component becomes negligible. So, the residual shift at transit is mainly due to the motion of the Moon itself from apogee to perigee.

Figure 10: Here, the Moon just crossed the equator plane at its ascending node and is on its way towards the apogee almost at "fu1l speed". In that case, the frequency down shift looks like a sinewave, with its extrema slightly after moonrise, and slightly before moonset. This is due to the fact that both the Moon and the EME station are located "above" the equator plane. This reverse effect increases when the declination increases further north. Here the whole transit shift is shared between radial speed and declination motion components.

Figure 11: Here, the Moon is close to its northernmost declination and on its way towards the perigee roughly at "half speed". In that case, the frequency shift does show a real sinusoidal aspect, from moonrise to moonset. As quoted above, the right ascension speed component, as well as that due to the earth rotation are both null at transit. The declination being almost stationary, its motion component becomes negligible. So, the residual shift at transit is essentially due to the motion of the Moon from apogee to perigee. The confirmation of the reverse shift is clearly shown here. The shift extrema occur roughly two hours after moonrise and two hours before moonset. This rather surprising result (well observed by the author on 1296 MHz) is due to the earth rotation speed component on the axis Station−Moon. This time, the shift becomes null half an hour after transit, the Moon being now on its way towards perigee.

Figure 12: Here, the Moon is very close to its descending node and on its way towards the perigee roughly at one third of the maximum radial speed. In that case, the frequency down shift still shows a limited sinusoidal aspect, with a slight extremum at moonrise, because of the small northern declination, and no extremum anymore at moonset, the declination being then southern. This time, the shift becomes null after transit, for the same reason as above. At transit, the Moon is approaching its descending node, and so, moving away from the EME station. The shift moves "downwards" accordingly, but, the Moon being on its way towards perigee, the final overall shift still remains positive,

Note: On the plots, the table time marks written in *italics* refer to the previous or next date according to rise, transit and set times of date.

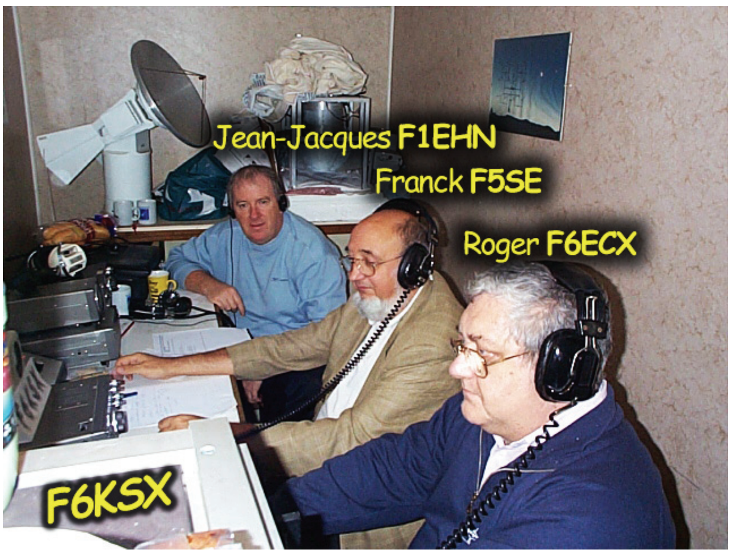
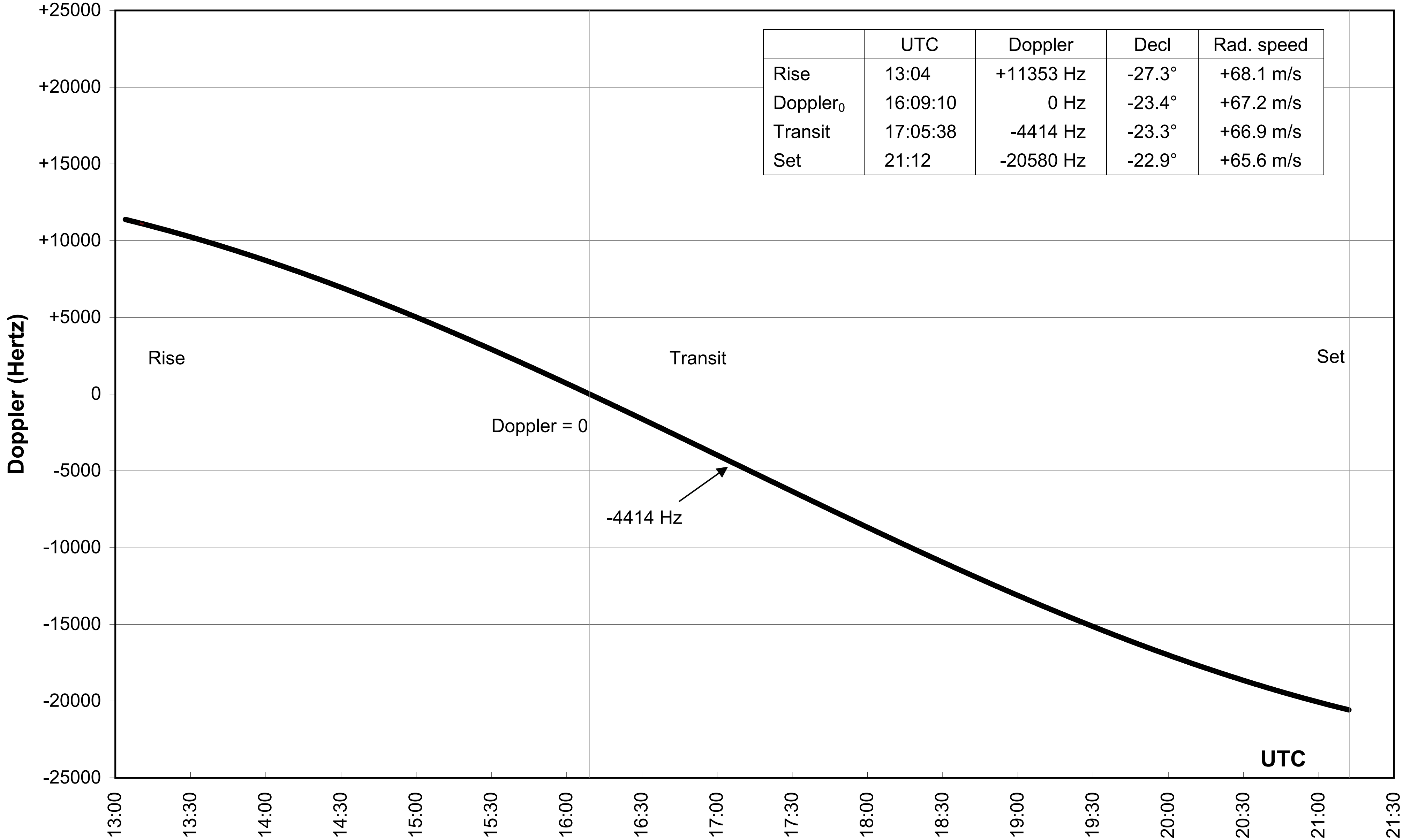


Figure 9

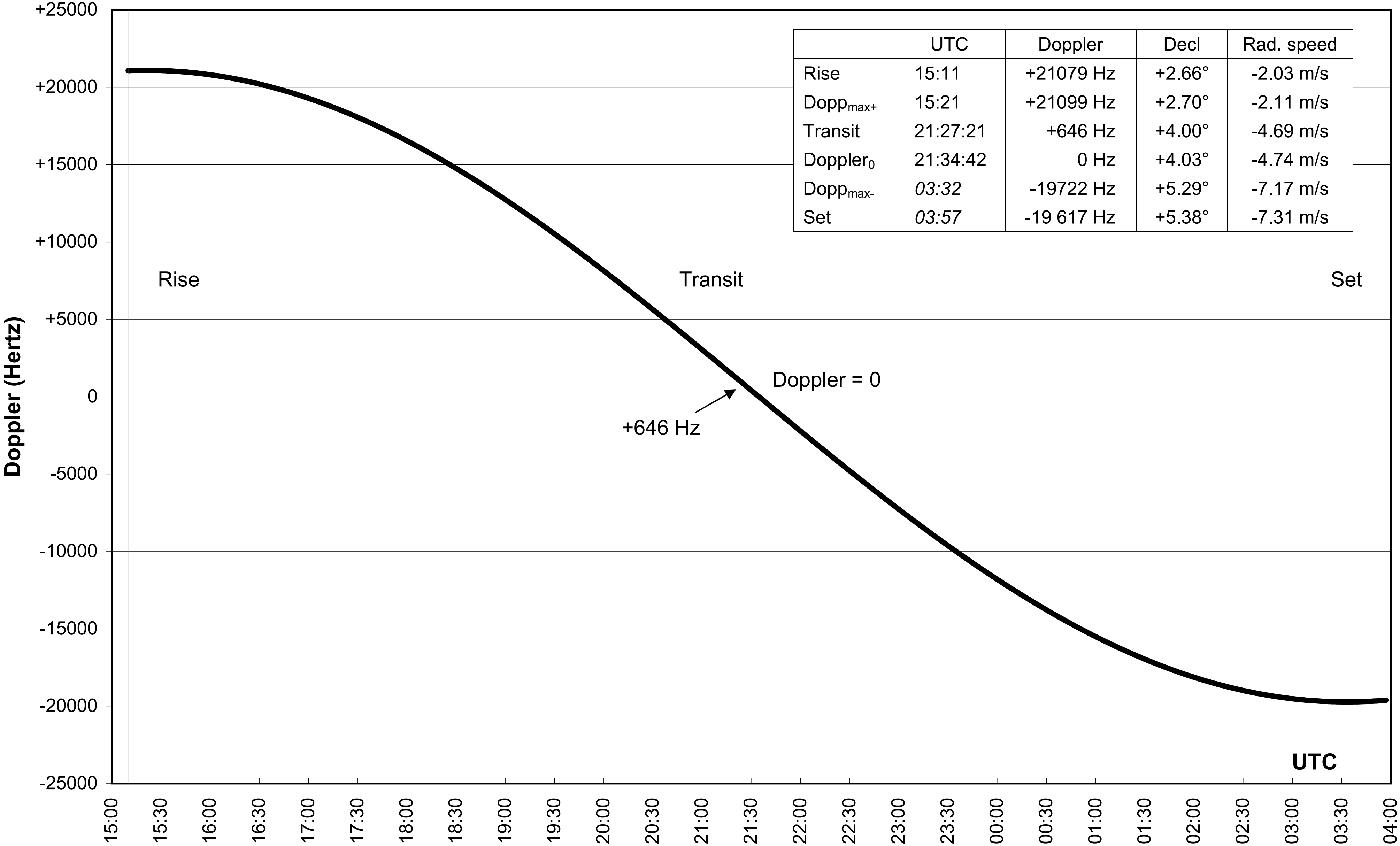
10 GHz F6KSX (JN18AR) Doppler plot, on 2002 November 10



	UTC	Doppler	Decl	Rad. speed
Rise	13:04	+11353 Hz	-27.3°	+68.1 m/s
$Doppler_0$	16:09:10	0 Hz	-23.4°	+67.2 m/s
Transit	17:05:38	-4414 Hz	-23.3°	+66.9 m/s
Set	21:12	-20580 Hz	-22.9°	+65.6 m/s

Figure 10

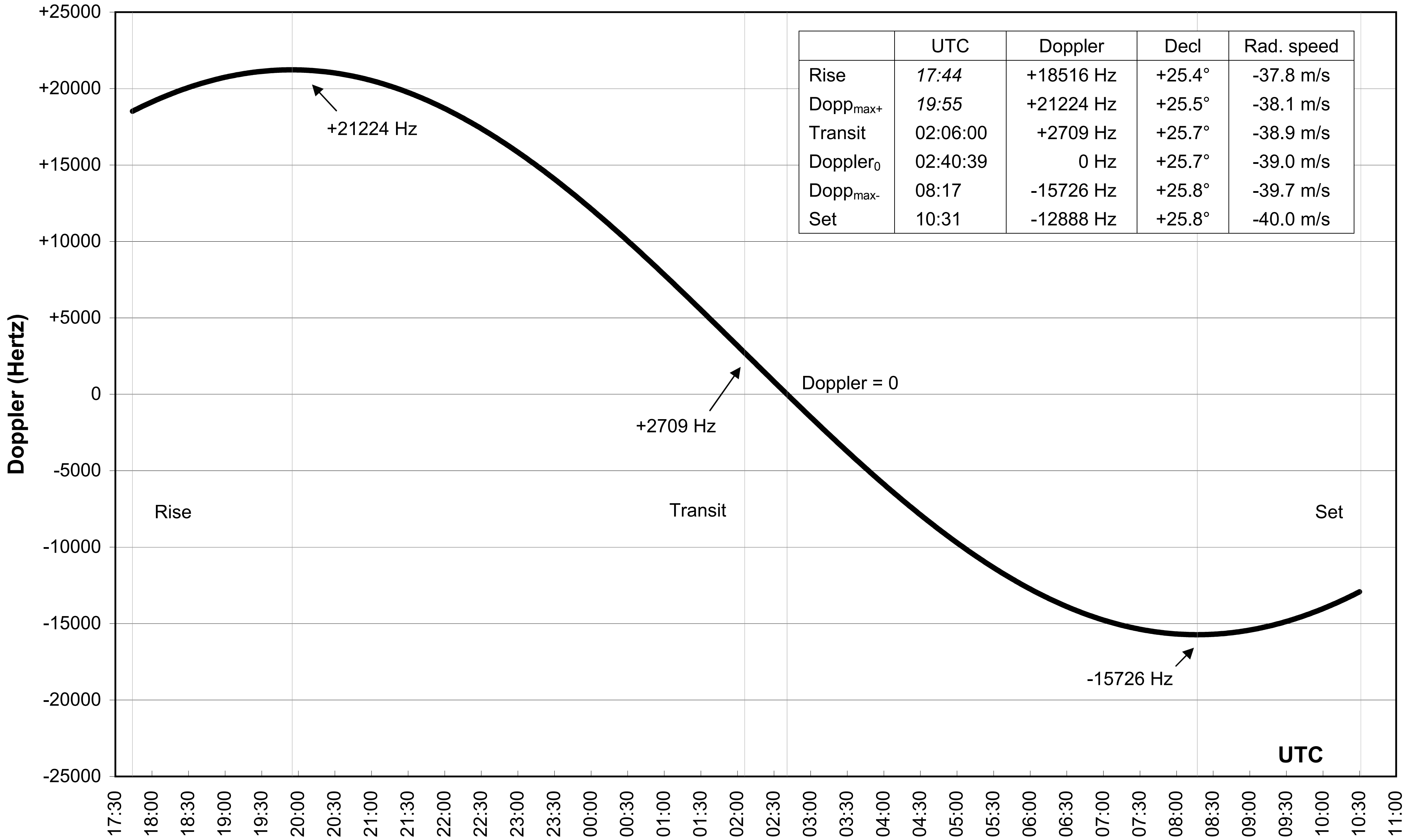
10 GHz F6KSX (JN18AR) Doppler plot, 2002 November 16 / 17



	UTC	Doppler	Decl	Rad. speed
Rise	15:11	+21079 Hz	+2.66°	-2.03 m/s
$Dopp_{max+}$	15:21	+21099 Hz	+2.70°	-2.11 m/s
Transit	21:27:21	+646 Hz	+4.00°	-4.69 m/s
$Doppler_0$	21:34:42	0 Hz	+4.03°	-4.74 m/s
$Dopp_{max-}$	*03:32*	-19722 Hz	+5.29°	-7.17 m/s
Set	*03:57*	-19 617 Hz	+5.38°	-7.31 m/s

Figure 11

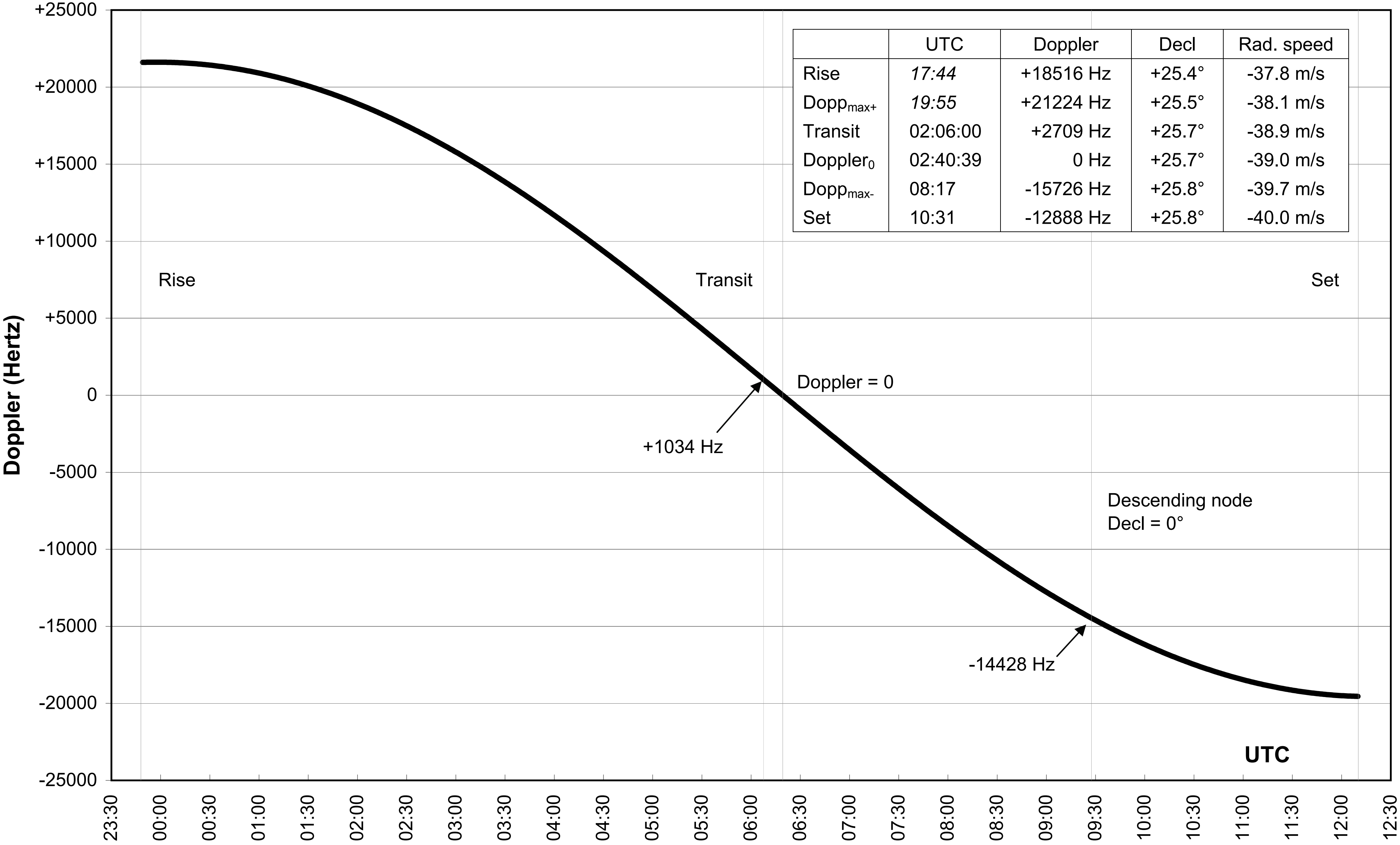
10 GHz F6KSX (JN18AR) Doppler plot, 2002 November 22 / 23



	UTC	Doppler	Decl	Rad. speed
Rise	*17:44*	+18516 Hz	+25.4°	-37.8 m/s
$Dopp_{max+}$	*19:55*	+21224 Hz	+25.5°	-38.1 m/s
Transit	02:06:00	+2709 Hz	+25.7°	-38.9 m/s
$Doppler_0$	02:40:39	0 Hz	+25.7°	-39.0 m/s
$Dopp_{max-}$	08:17	-15726 Hz	+25.8°	-39.7 m/s
Set	10:31	-12888 Hz	+25.8°	-40.0 m/s

Figure 12

10 GHz F6KSX (JN18AR) Doppler plot, 2002 December 26 / 27



	UTC	Doppler	Decl	Rad. speed
Rise	*17:44*	+18516 Hz	+25.4°	-37.8 m/s
$Dopp_{max+}$	*19:55*	+21224 Hz	+25.5°	-38.1 m/s
Transit	02:06:00	+2709 Hz	+25.7°	-38.9 m/s
$Doppler_0$	02:40:39	0 Hz	+25.7°	-39.0 m/s
$Dopp_{max-}$	08:17	-15726 Hz	+25.8°	-39.7 m/s
Set	10:31	-12888 Hz	+25.8°	-40.0 m/s